Two methods of simultaneous analysis in
Common components and unit weights

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Reprinted from
ICASTOR Journal of Mathematical Sciences
Vol. 3, N° 2 (2009)
TWO METHODS OF SIMULTANEOUS ANALYSIS IN COMMON COMPONENTS AND UNIT WEIGHTS

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ABSTRACT

Analysis of common components and specific weights are two methods which allow simultaneous analysis of multiple tables paired line wise. One edge thus determines common components and specific weights for all tables. This method has been applied in areas such as ecology. It has also been used in coupling problems of many measuring instruments. To make it more changeable, new properties and new algorithm have recently been come forth. However, to find the orthogonal common components, successive approach is used so that once the first solution has been found, deflations are made in each table. This long strategy has been used to construct orthogonal bases, having the box of MCOA. Here, one has to find these common components simultaneously, determining iterative at once through year process, orthogonal year matrix containing these common components.

KEYWORDS: Common components and specific weights analysis, multiple data sets, IDIOSCAL.

The Analysis in Common Components and Unit Weights of Qannari et al. (2000 and 2001) is a method to be classified among the methods of analysis of the data of the multiple tables. It is based on the minimization of several criteria rising from various models.

To analyze the tables $X_k$ of dimension $n \times m_k$ for $k = 1, ..., K$, models of analysis of the data were built starting from the scalar matrices of products $W_k = X_k X_k^t$ associated with the tables $X_k$ (for example Statis (L’Hermier des Plantes), 1976; Lavit, 1988), the Multiple factorial Analysis (Escofier and Pages, 1988), and the ACOM (Chessel and Hanafi, 1996). These tables were paired in lines (bearing on the same individuals) and able to be centred and possibly standardized.

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Thus, in connection with the Analysis of Common Components and unit Weights, it was proposed to have a certain number of the models. The first model stipulates the equality of the matrices if the scalar products \( W_k = W + E_k \) (see Hanafi and Qannari (2008)). To determine a compromised configuration, a matrix \( W \) is required of kind with minimized

\[
\sum_{k=1}^{k} \left\| W_k - W \right\|^2 = \sum_{k=1}^{k} \text{trace}((W_k - W)'(W_k - W))
\]

The resolution of this problem leads to the solution \( W = \frac{1}{k} \sum_{k=1}^{k} W_k \). This matrix is semi-definite positive since average of matrices of the scalar products \( W_k \) are semi-definite positive. This compromised configuration is to be brought closer with those found in Statis (L’Hermier des Plantes, 1976, Lavit, 1988) and in the Multiple Factorial Analysis (Lavit, 1988) by Analysis of principal components in the table obtained by juxtaposition of the tables \( X_k (k = 1, \ldots, K) \). The second model is defined by \( W_k = \alpha_k W + E_k \). This resulted in determining the coefficients \( \alpha_k (k = 1, \ldots, K) \) and a matrix \( W \) in order to minimize the following quantity

\[
\sum_{k=1}^{k} \left\| W_k - \alpha_k W \right\|^2
\]

By forcing the coefficients \( \alpha_k \) of the kind \( \sum_{k=1}^{k} \alpha_k^2 = 1 \), one can in this manner find the compromise determined by the Statis method.

The third model is ACCPS. It stipulates the existence of common components for all the configurations, but the weights of the configurations on these components for all the configurations, but the weights of the configurations on these components can be different. This model can be formulated in the following manner \( W_k = QD_k Q' + E_k = \sum_{s=1}^{n-1} \lambda_s^{(k)} q_s q_s' + E_k \), where \( n \) is the number of the individuals, \( Q \) is an orthogonal matrix whose columns are the common components \( (q_s) (s = 1, \ldots, n-1) \), the number of these common components is mostly equal to \( n-1 \) because of the centering of the tables \( X_k \), and \( D_k \) is a diagonal matrix containing on its principal diagonal the scalars noted \( \lambda_s^{(k)} (s = 1, \ldots, n-1) \) called unit weights associated the common components.

The determination of the common components and the associated unit weights is led in a sequential way and is done in a successive way. Thus, at stage 1, a first normalized component \( q_1 \) is given as well as the weights associated by the minimization of the function of loss as follows

\[
L_1 = \sum_{k=1}^{k} \left\| W_k - \lambda_1^{(k)} q_1 q_1' \right\|^2
\]

For the scalars \( \lambda_1^{(k)} (k = 1, \ldots, K) \) fixed, the optimal vector which minimizes \( L_1 \) is clean vector normalized of \( \sum_{k=1}^{k} \lambda_1^{(k)} W_k \) associated with the greatest eigenvalue. When \( q_1 \) is fixed, the optimal unit weights are given by \( \lambda_1^{(k)} = q_1' W_k q_1 \). An iterative algorithm of Qannari et al. (2000) makes it possible to determine the common component and the optimal weights which are associated with them. Hanafi and Qannari (2008) show that this common component is solution of the problem as follows.
To maximize \( \sum_{k=1}^{k} (q'W_kq) \) under the constraint \( \|q\| = 1 \) \( \text{(4)} \)

To determine the components of order \( s \geq 2 \), one minimizes the function as follows

\[
L_s = \sum_{k=1}^{k} \left| \left| W_k^{(s)} - \lambda_s^{(k)}qq' \right| \right|^2
\]

under the constraint \( \|q\|^2 = 1 \) where \( W_k^{(s)} = X_k'X_k \) is the matrix of scalar products with \( X_k^{(s)} = X_k - \sum_{j<i} q_jq_j'X_k \). It is also shown that the optimal vector which minimizes \( (5) \) is normalized eigen vector \( \sum_{k=1}^{k} \lambda_s^{(k)}W_k^{(s)} \) where the unit weights are determined \( \lambda_s^{(k)} = q_s'W_s^{(s)}q_s = q_s'W_kq_s \).

The criterion \( (5) \) being the constraint of standard on the common component is equivalent to the criterion \( (4) \) where \( W_k \) is replaced by \( W_k^{(s)} \).

Lazraq et al. (1992), Kiers et al. (1994) developed an alternative of the generalised canonical analysis of Carroll (1968) simultaneously based on coefficient RV (Escoufier, 1973) to extend the canonical analysis of Hotelling (1936). They seek to maximize compared to a compromise \( Q \) and with a matrix \( A_k \) a matrix of weight associated with the table \( X_k \) the quantity

\[
T(Q,A_1,...,A_k) = \sum_{k=1}^{k} w_kRV^2(Q,X_kA_k)
\]

under the constraint \( Q'Q = I_r \), where \( w_k \) are weightings associated with the tables \( X_k \).

For \( Q \) fixed, the maximum value of \( RV^2(Q,X_kA_k) \) is reached when \( A_k = P_kQ(P_k = X_k(X_k'X_k)^{-1}X_k' \) is the projector under space generated by the columns of \( X_k \) is given by an expression which does not depend any more on \( Q \) and on \( P_k \) (for a more detailed demonstration of this proof see Lazraq et al. (1992) and Kiers et al. (1994)). Lastly, by making some transformations, the upper limit of \( (6) \) is the quantity to maximize as follows:

\[
T(Q) = \sum_{k=1}^{k} w_k\text{trace}(Q'P_kQ)^2
\]

under the constraint \( Q'Q = I_r \). \( \text{(7)} \)

The criterion \( (7) \) can be obtained by method IDIOSCAL (Carroll and Chang, 1972). IDIOSCAL minimizes on arbitrary matrices \( Q(n \times r) \) and \( C_k (r \times r) \) the function

\[
F(Q,C_1,...,C_k) = \sum_{k=1}^{k} \left| \left| S_k - QC_kQ' \right| \right|^2
\]

for \( S_1,...,S_k \) symmetrical matrices \( n \times n \) given. But these matrices \( S_k = w_k^{1/2}P_k \) can have an instability in their calculation because of metric of Mahalanobis in the projectors (case where the variables of the tables \( X_k \) are correlated). For this reason we substitute the \( P_k \) projectors with the scalar matrices of products \( W_k \) of the tables \( X_k \) in the criterion \( (7) \). The advantage of the scalar matrices of products between individuals is not any more to show. Chessel and Hanafi (1996) also used these operators to define the ACOM contrary to the generalized canonical analysis of Carroll (1968) who used the projectors. The article is organized in the following way: in Section...
1, we state the two allowing to dilute the successive solutions to the simultaneous solutions. Lastly, in Section 2, we give the associated algorithms which make it possible to determine the simultaneous solutions. Thus the data which are analyzed present in the following manner:

\[
X = \begin{bmatrix} X_1 | X_2 | ... | X_k \end{bmatrix} \text{ a table of dimension concatenate of the tables } X_k \text{ of dimension } n \times m_k
\]

where \( m = \sum_{k=1}^{k} m_k \). \( r \) is the number of the eigenvalues of \( \sum_{k=1}^{k} W_k \) strictly positive. \( A' \) indicate transposition of the square matrix \( A \), and \( diag(A) \) indicates the diagonal matrix of diagonal \( A \). \( I_n \) indicate the matrix identify of order \( n \).

1. SIMULTANEOUS CRITERIA OF THE ANALYSIS IN COMMON COMPONENTS AND UNIT WEIGHTS

To determine the simultaneous solutions of the analysis in common components and unit weights, two criteria are proposed. The first is a pseudo criterion since it does not give the same function exactly to be maximized when the number of columns of \( Q \) is strictly higher than one. On the other hand, the second criterion is really the simultaneous form of the ACCPS since the function is found same in the matrix \( Q \).

Consequently, the first criterion is the pseudo Simultaneous Analysis of Common Components and Unit Weights (PASCSCP) which is formulated in the following manner:

To Maximize : \( G(Q) = \sum_{k=1}^{k} \text{trace}(Q'W_k Q)^2 \) \hspace{1cm} (9)

under the constraint \( Q'Q = I_r \), since the criterion (4) is found when the matrix \( Q \) is reduced to only one column. In (9), one does not take account of weightings \( w_k \), but that does not harm the general information. It is noticed that the function \( G(Q) \) can be put in the form \( G(Q) = \text{trace}Q'\left(\sum_{k=1}^{k} W_k QQ'W_k Q\right) \).

On the other hand, the second criterion relates to the Simultaneous Analysis in Common Components and Unit Weights (ASCCPS) which is stated in the following manner:

To maximize : \( H(Q) = \sum_{k=1}^{k} \text{trace}[Q'W_k diag(Q'W_k Q)] \) \hspace{1cm} (10)

under the constraint \( Q'Q = I_r \), because the criterion (4) is also found if the matrix \( Q \) is reduced to only one column. If one notes \( Q = \{q_1,...,q_s,...,q_r\} \), the matrix which is formed by the vector columns \( q_s \), the criterion (10) can be still written in an equivalent way:

To maximize : \( H(q_1,...,q_r) = \sum_{k=1}^{k} \sum_{s=1}^{s'} (q_s W_k q_s')^2 = \sum_{k=1}^{k} \sum_{s=1}^{s'} \left[ \lambda_s^{(k)} \right]^2 \) \hspace{1cm} (11)

under constraints \( \|q_s\| = 1 \) for all \( s = 1,...,r \), where \( \lambda_s^{(k)} = q_s' W_k q_s \) are the unit weights associated with the components common to the tables \( X_k \) for \( k = 1,...,K \). By adopting the new formulation of the ACCPS of Hanafi and Qannari (2008), the criterion (11) consists of determining common components \( q_s \) as well as the partial components \( c_i^{(k)} = X_k u_i^{(k)} \) so as to maximize the quantity.
TWO METHODS OF SIMULTANEOUS ANALYSIS IN COMMON COMPONENTS AND UNIT WEIGHTS

\[ H\left(q_1,\ldots,q_r,u_{1}^{(k)},\ldots,u_{r}^{(k)}\right) = \sum_{k=1}^{K} \sum_{r=1}^{R} \text{cov}^4\left(X_{r}u_{r}^{(k)},q_{s}\right) \]  

(12)

where the unit weights associated with the components \(q_{s}\) are given by the numbers \(\lambda_{s}^{(k)} = \text{cov}^2(X_{r}u_{r}^{(k)},q_{s})\), with \(u_{r}^{(k)}\) is the normalized vector relating to the table \(X_{k}\).

2. THE ALGORITHMS OF PASCCPS AND ASCCPS

In the article, we follow the steps of construction of the algorithms developed in Kiers et al. (1994) and in Lafosse and Ten Berge (2006) which are very similar. Besides, in our paper, we substitute the matrices \(S_{k} = w_{k}^{1/2} P_{k}\) with the matrices of the scalar products \(W_{k}\) for PASCCPS on the one hand, and in the ASCCPS we have instead of criterion of Lafosse and Ten Berge (2006) on the other. But here on the latter point, it is a synthesis of the two steps (Kiers et al. and of Lafosse and Ten berge), which makes the difference on the level for the proof of monotony.

a) The algorithm of PASCCPS

\(Q\) is initialized by \(Q^{(0)}\) (for example selected orthogonal matrix by chance). Then \(Q^{(0)}\) is updated by the matrix \(Q^{(1)}\), \(Q^{(1)}\) by \(Q^{(2)}\), etc. In general, the current matrix is updated by matrix

\[ Q^{u} = UV' \]  

(13)

where \(U\) and \(V\) are the matrices determined by the singular value decomposition of the matrix (Cliff, 1966).

\[ \sum_{k=1}^{K} W_{k}QQ'W_{k}Q = UDV' \]  

(14)

with \(U'U = V'V = I_{r}\) and \(D\) is a diagonal matrix whose elements of the diagonal are positive or null. This procedure is repeated until convergence. After having determined the update of \(Q\), it is a question of establishing the monotony of the algorithm, i.e., to show that the procedure makes to grow monotonically the function

\[ G(Q) = \sum_{k=1}^{K} \text{trace}(Q'W_{k}Q)^2 \]  

or, \(G(Q^u) \geq G(Q)\)

To show this inequality, we develop the following inequality:

\[ \left\|W_{k}^{1/2}Q^{(u)}W_{k}^{1/2} - W_{k}^{1/2}Q^{u}Q^{(u)}W_{k}^{1/2}\right\|^2 \geq 0 \]  

(15)

or

\[ \sum_{k=1}^{K} \text{trace}(Q'W_{k}Q)^2 + \sum_{k=1}^{K} \text{trace}(Q^{u}W_{k}Q^{u})^2 \geq 2 \sum_{k=1}^{K} \text{trace}(Q'W_{k}Q)(Q'W_{k}Q^{u}) \]  

(16)

and from
\[ \| Q'W_kQ - Q''W_kQ \|^2 \geq 0 \]  

(17)

results the following inequality:

\[ \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ') \geq 2 \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ') - \sum_{k=1}^{k} \text{trace}(Q''W_kQ)^2 \]  

(18)

Finally, the choice of (13) like the update of \( Q \) enables us to write (Cliff, 1966)

\[ \sum_{k=1}^{k} \text{trace}Q''(W_kQQ'W_kQ) \geq \sum_{k=1}^{k} \text{trace}Q'(W_kQQ'W_kQ) = G(Q) \]  

(19)

While rewriting (16) as

\[ G(Q'') = \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ') \geq 2 \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ') - G(Q) \]  

(20)

and while combining (20) with (18), we find

\[ G(Q'') = 2 \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ') - G(Q) \geq \]  

\[ 4 \sum_{k=1}^{k} \text{trace}(Q''W_kQ)(Q'W_kQ) - 3G(Q) \]  

(21)

Finally, by combining (21) with (19), we obtain the inequality

\[ G(Q'') \geq 4G(Q) - 3G(Q) = G(Q) \]

Thus the update defined in (13) is the proof of growing the function monotonically \( G(Q) \).

The procedure of determination of described solution by Ci-high is summarized by the following algorithm:

1. To Choose for example the initial matrix \( Q \) which is consisted of the first clean vectors columns \( r \) of the matrix \( \sum_{k=1}^{k} W_k \) and \( \epsilon \) (example, 0.00001).
2. To calculate \( \sum_{k=1}^{k} W_kQQ'W_kQ \)
3. To make the singular value decomposition of \( \sum_{k=1}^{k} W_kQQ'W_kQ = UUV' \).
4. Update the matrix \( Q \) by the matrix \( Q'' = UV' \)
5. If \( G(Q'') - G(Q) \leq \epsilon \), the algorithm converged; if not, \( Q = Q'' \). In this case outward journey is 2.

After having found the solution, i.e. the matrix \( Q \) having the vector columns \( q_s = (s = 1, \ldots, r) \) one can, for each table and each common component \( q_s \) calculate the unit weights \( \lambda_{s}^{(k)} = q_sW_kq_s \) \((s = 1, \ldots, r \text{ and } k = 1, \ldots, K) \) which are contained in the matrix \( Q'W_kQ \).

b) The algorithm of the ASCCPS

By taking into account the adopted initialization with a), we can build be update of \( Q \) by the matrix.
$Q'' = UV'$

where $U$ and $V$ are matrices as $U'U = V'V = I$, obtained by the singular value decomposition of the matrix:

$$\sum_{k=1}^{k} W_kQ_{\text{diag}}(Q'W_kQ) = UDV'$$

This procedure of update is also repeated until convergence. Thus after having found the update of $Q$, one can now establish the monotony of the algorithm, i.e., to show that the procedure makes the following function grow monotonically:

$$H(Q) = \sum_{k=1}^{k} \text{trace}[Q'W_kQ_{\text{diag}}(Q'W_kQ)]$$

or $H(Q') \geq H(Q)$. To establish this relation, we consider the $s^{th}$ diagonal element of the matrix $Q''W_kQ_{\text{diag}}(Q'W_kQ)$ definie par:

$$q_i^sW_kq_i^s, q_i^sW_kq_i^s = q_i^sG_{ks}q_s$$

where $G_{ks} = W_kq_i^sW_k$ is a positive semi-definie matrix and $q_i$ is the $s^{th}$ vector column of the matrix $Q$.

From the inequality

$$\|G_{ks}q_i - G_{ks}q_i\|^2 \geq 0$$

we obtain the following inequality:

$$q_i^sG_{ks}q_s + q_i^sG_{ks}q_i^s \geq 2q_i^sG_{ks}q_s$$

While taking the sum of (26) on all the diagonal elements of the matrix $Q''W_kQ_{\text{diag}}(Q'W_kQ)$, and taking the sum of $k$, we obtain the following inequality:

$$\sum_{k=1}^{k} \text{trace}(Q'W_kQ_{\text{diag}}(Q'W_kQ)) \geq \sum_{k=1}^{k} \text{trace}(Q''W_kQ_{\text{diag}}(Q'W_kQ))$$

The update of checks according to Cliff (1966) is the following inequality:

$$\sum_{k=1}^{k} \text{trace}(Q''W_kQ_{\text{diag}}(Q'W_kQ)) \geq \sum_{k=1}^{k} \text{trace}(Q'W_kQ_{\text{diag}}(QW_kQ)) = H(Q)$$

Then, using the following inequality

$$\|W_k^{1/2}q_i^sW_k^{1/2} - W_k^{1/2}q_i^sW_k^{1/2}\|^2 \geq 0$$

one finds the relation:

$$q_i^sW_kq_i^s(q_i^sW_kq_i^s) + q_i^sW_kq_i^s(q_i^sW_kq_i^s) \geq 2q_i^sW_kq_i^s(q_i^sW_kq_i^s)$$
While summoning (30) on the $s^{th}$ diagonal element of the matrices $Q'W_kQ \text{diag}(Q'W_kQ)$, $Q''W_kQ' \text{diag}(Q''W_kQ'')$, $Q''W_kQ \text{diag}(Q''W_kQ'')$ and on $k$, we obtain the following relation:

$$
\sum_{k=1}^{k} \text{trace}(Q'W_kQ \text{diag}(Q'W_kQ)) + \sum_{k=1}^{k} \text{trace}(Q''W_kQ' \text{diag}(Q''W_kQ'')) \geq 2\sum_{k=1}^{k} \text{trace}(Q''W_kQ' \text{diag}(Q''W_kQ'))
$$

(31)

From inequality (31), one may find

$$
H(Q'') = \sum_{k=1}^{k} \text{trace}(Q''W_kQ'' \text{diag}(Q''W_kQ'')) \geq 2\sum_{k=1}^{k} \text{trace}(Q''W_kQ'' \text{diag}(Q''W_kQ'')) - H(Q)
$$

(32)

By rewriting (27) in the following manner

$$
\sum_{k=1}^{k} \text{trace}(Q''W_kQ'' \text{diag}(Q''W_kQ'')) \geq 2\sum_{k=1}^{k} \text{trace}(Q''W_kQ'' \text{diag}(Q''W_kQ'')) - H(Q)
$$

(33)

and by combining the relations (28), (32) and (33), we obtain the following required result:

$$
H(Q'') \geq 4H(Q) - 3H(Q) = H(Q)
$$

(34)

from where the monotony of the algorithm is established.

The summary of the algorithm described above is given similarly by the following procedure:

1. The Choose, for example, the initial solution $Q$ which is the solution of the preceding algorithm and $\varepsilon$ (example, 0.00001)
2. To calculate $\sum_{k=1}^{k} W_kQ \text{diag}(Q'W_kQ)$
3. To make the singular value decomposition of $\sum_{k=1}^{k} W_kQ \text{diag}(Q'W_kQ) = UDV'$
4. To update the matrix $Q$ by the matrix $Q'' = UV'$
5. If $H(Q'') - H(Q) \leq \varepsilon$, the algorithm is converged; if not, then $Q = Q''$. In this case outward journey is 2.

The maximization of (11) compared to the vectors under the constraints of standard leads to the equations with the clean ones:

$$
\sum_{k=1}^{k} \lambda_{k}^{(s)} W_k q_s = \mu_s q_s , \quad s = 1, \ldots, r
$$

(35)

The vectors (common components) and unit weights of (35) can be given for each value of be $s$ by the algorithm of Qannari et al. (2000).
3. APPLICATION

In this paragraph we will have the numerical results of these two simultaneous methods and will make a comparison with the successive approach of Hanafi and Qannari (2008).

3.1- PRESENTATION OF THE DATA

To establish the link with the ACCPS, we take again the data file in ididactic matter which was proposed by Williams and Lagron (1984), taken again by (Hanafi and Kiers (2006) and Hanafi and Qannari (2008)). These data relate to the appreciation of the wines with judges.

A jury made up of four judges evaluated the appearance of eight wines according to the procedure known as free profile where each judge notes on a scale going from 0 to 10 the products according to his own descriptors (variables). For a product having a given descriptor, the note allotted by a judge corresponds to the intensity which it perceives and which it is able, thanks to a preliminary drive, to translate in the form of a note. The data are reproduced in appendix.

3.2- PRESENTATION OF THE RESULTS

Table 1 gives the unit weights and the percentages of inertia of the various tables restored by the common components in the successive method (see Hanafi and Qannari, 2008)).

In the same way, in Tables 2 and 3, one finds the unit weight and the percentages of inertia of the two simultaneous methods suggested in this paper. The inertia of each table was brought back to 1, each unit weight can be perceived like a percentage of inertia restored by each common component, as it is the case in Hanafi and Qannari (2008).

<table>
<thead>
<tr>
<th>Dimension</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge 1</td>
<td>Unit Weights</td>
<td>0.670</td>
<td>0.010</td>
<td>0.190</td>
<td>0.080</td>
<td>0.020</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.670</td>
<td>0.680</td>
<td>0.870</td>
<td>0.950</td>
<td>0.970</td>
<td>0.980</td>
</tr>
<tr>
<td>Judge 2</td>
<td>Unit Weights</td>
<td>0.660</td>
<td>0.040</td>
<td>0.220</td>
<td>0.030</td>
<td>0.010</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.660</td>
<td>0.700</td>
<td>0.920</td>
<td>0.950</td>
<td>0.960</td>
<td>0.990</td>
</tr>
<tr>
<td>Judge 3</td>
<td>Unit Weights</td>
<td>0.780</td>
<td>0.060</td>
<td>0.110</td>
<td>0.010</td>
<td>0.020</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.780</td>
<td>0.840</td>
<td>0.950</td>
<td>0.960</td>
<td>0.980</td>
<td>0.990</td>
</tr>
<tr>
<td>Judge 4</td>
<td>Unit Weights</td>
<td>0.210</td>
<td>0.470</td>
<td>0.280</td>
<td>0.030</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.210</td>
<td>0.680</td>
<td>0.960</td>
<td>0.990</td>
<td>1.000</td>
<td>1.000</td>
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</tbody>
</table>

Table 1. Unit weights, percentages of inertias of the various tables restored by the common components dimension (method : ACCPS)
Table 2. Specific weight (method : PASCCPS)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Unit Weights</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge 1</td>
<td>Unit Weights</td>
<td>0.606</td>
<td>0.175</td>
<td>0.073</td>
<td>0.103</td>
<td>0.024</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.606</td>
<td>0.782</td>
<td>0.855</td>
<td>0.958</td>
<td>0.983</td>
<td>0.995</td>
<td>1.000</td>
</tr>
<tr>
<td>Judge 2</td>
<td>Unit Weights</td>
<td>0.672</td>
<td>0.218</td>
<td>0.015</td>
<td>0.037</td>
<td>0.005</td>
<td>0.036</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.672</td>
<td>0.891</td>
<td>0.906</td>
<td>0.944</td>
<td>0.950</td>
<td>0.986</td>
<td>1.000</td>
</tr>
<tr>
<td>Judge 3</td>
<td>Unit Weights</td>
<td>0.712</td>
<td>0.221</td>
<td>0.018</td>
<td>0.002</td>
<td>0.024</td>
<td>0.004</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.712</td>
<td>0.933</td>
<td>0.952</td>
<td>0.954</td>
<td>0.979</td>
<td>0.983</td>
<td>1.000</td>
</tr>
<tr>
<td>Judge 4</td>
<td>Unit Weights</td>
<td>0.309</td>
<td>0.205</td>
<td>0.439</td>
<td>0.030</td>
<td>0.002</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.309</td>
<td>0.515</td>
<td>0.955</td>
<td>0.985</td>
<td>0.988</td>
<td>0.992</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 3. Specific weight (method : ASCCPS)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Unit Weights</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Judge 1</td>
<td>Unit Weights</td>
<td>0.623</td>
<td>0.174</td>
<td>0.050</td>
<td>0.108</td>
<td>0.031</td>
<td>0.006</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.623</td>
<td>0.797</td>
<td>0.847</td>
<td>0.956</td>
<td>0.988</td>
<td>0.994</td>
<td>1.000</td>
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<tr>
<td>Judge 2</td>
<td>Unit Weights</td>
<td>0.642</td>
<td>0.241</td>
<td>0.022</td>
<td>0.029</td>
<td>0.004</td>
<td>0.045</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.642</td>
<td>0.883</td>
<td>0.906</td>
<td>0.935</td>
<td>0.939</td>
<td>0.984</td>
<td>1.000</td>
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<tr>
<td>Judge 3</td>
<td>Unit Weights</td>
<td>0.732</td>
<td>0.193</td>
<td>0.029</td>
<td>0.002</td>
<td>0.017</td>
<td>0.007</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>% cumulated Inertia</td>
<td>0.732</td>
<td>0.926</td>
<td>0.955</td>
<td>0.958</td>
<td>0.975</td>
<td>0.983</td>
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</tr>
<tr>
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<td>Unit Weights</td>
<td>0.257</td>
<td>0.241</td>
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<td>0.004</td>
<td>0.003</td>
<td>0.007</td>
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<tr>
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<td>% cumulated Inertia</td>
<td>0.257</td>
<td>0.499</td>
<td>0.957</td>
<td>0.985</td>
<td>0.989</td>
<td>0.992</td>
<td>1.000</td>
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CONCLUSION

They methods developed in this paper make it possible to determine the simultaneous solutions. They calculate local solutions. To obtain total solutions, it is recommended to start from several initializations and, to adopt the best solutions. It is noticed that these simultaneous algorithms are fast. The unit weights which are obtained resemble perfectly with those found in the successive approach. It is seen well that judge 4 has a behavior different from the other judges for the three approaches. Apart from judge 4, inertias of the successive method are larger for the first dimension compared to those of the simultaneous methods; on the other hand, cumulated inertias of the first two dimensions of these two simultaneous methods are a little higher than those of the successive method. All these methods have a common objective; one can thus indifferently use one of the methods to treat these kinds of the data.
REFERENCES

1. Carroll, J.D.A generalization of canonical correlation analysis to three or more sets of variables; Proceeding of the 76th Convention of the American Psychological association; 3, 1968, pp.227-228
Annexure : data

$$x_1$$  | red | gilded | soft | plum |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
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<td>0</td>
<td>5</td>
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<td>6</td>
<td>3</td>
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<td>V3</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>5</td>
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<tr>
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<td>7</td>
<td>4</td>
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</tbody>
</table>

$$x_2$$  | ruby | coloured | Intensity |
<table>
<thead>
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<td>3</td>
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</tr>
<tr>
<td>V4</td>
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<tr>
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<tr>
<td>V8</td>
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<td>6</td>
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</table>

$$x_3$$  | red | blue | gilded | intensity |
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<th></th>
</tr>
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<td>2</td>
<td>5</td>
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<td>0</td>
<td>6</td>
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<td>5</td>
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</tr>
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<td>V7</td>
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<td>4</td>
<td>3</td>
</tr>
<tr>
<td>V8</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>5</td>
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</tbody>
</table>

$$x_4$$  | deep | expenses | brilliant |
<table>
<thead>
<tr>
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</thead>
<tbody>
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</tr>
<tr>
<td>V2</td>
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</tr>
<tr>
<td>V3</td>
<td>10</td>
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<td>7</td>
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<tr>
<td>V4</td>
<td>7</td>
<td>7</td>
<td>8</td>
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<tr>
<td>V5</td>
<td>8</td>
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<td>V6</td>
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<tr>
<td>V7</td>
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<tr>
<td>V8</td>
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